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Participating media exposed to collimated short-pulse irradiation – A Laguerre–Galerkin solution

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Abstract

A new method is developed for the solution of radiative transfer in a one-dimensional absorbing and isotropically scattering medium with short-pulse irradiation on one of its boundaries. The time-dependent radiative intensity is expanded in a series of Laguerre polynomials with time as the argument. Moments of the radiative transfer equation, as well as of the boundary conditions, then yield a set of coupled time-independent radiative transfer problems. This set, in turn, is reduced to a set of algebraic equations by the application of the Galerkin method. The transient transmittance and reflectance of the medium are evaluated for various values of the optical thickness, scattering albedo and pulse duration. It is demonstrated that the Laguerre–Galerkin method is not only easier to implement and more efficient but also yields more accurate results compared to the direct application of the Galerkin method. The results are in very good agreement with those available in the literature.

Keywords: Laguerre–Galerkin method; Participating medium; Short-pulse irradiation; Transient radiative transfer

1. Introduction

Recent applications of transient radiative transfer in participating media involved extremely small time scales. Short-pulse lasers are being used to investigate properties of scattering and absorbing media in such applications as optical tomography, remote sensing, and combustion product analysis. In such applications, the time derivative of the radiation intensity in the radiative transfer equation cannot be neglected since its order of magnitude becomes comparable to that of the other terms in the equation.

Several techniques have been developed lately for the solution of transient radiative transfer problems in participating media. Kumar et al. [1,2] developed the two-flux, P_N and discrete ordinates approximate formulations. Mitra et al. [3] extended the P_1 approximation to two-dimensional enclosures. Kumar and Mitra [4,5] reviewed various approximate models, and developed regime maps for a proper model selection based on the specific application. Another application of the discrete ordinates method was performed by Sakami et al. [6] by implementing a high order upwind piecewise parabolic interpolation scheme to solve transient radiation in two-dimensional media. The integral formulation of the transient radiation, initially introduced by Pomraning [7], was improved and implemented by several researchers. Among those, Wu and Wu [8] developed the integral equation formulation for transient radiative transfer in a 3D absorbing and anisotropically scattering medium, and applied the formulation to solve the transient radiation in two-dimensional cylindrical, nonhomogeneous, absorbing and linearly scattering media [9]. Wu [10] also solved an exact integral equation formulation by an adaptation of the quadrature method. Another time-dependent integral formulation

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was developed by Tan and Hsu [11] who analyzed the wave propagation process inside the participating media. Later, they extended their one-dimensional integral equation model to three-dimensional geometry [12]. Guo and Kumar [13] presented a formulation by the radiation element method to solve transient radiative transfer with light radiation propagation effects in an inhomogeneous, scattering, absorbing, and emitting medium. A hybrid method based on the modified differential approximation suggested by Chandrasekhar [14] and the $P_{1/3}$ approximation was developed by Wu and Ou [15]. A three dimensional Monte Carlo simulation of transient radiative transfer, supported by an experimental study, has been performed by Guo et al. [16,17]. Rath et al. [18] extended the discrete transfer method to solve transient radiation problems in a participating medium subjected to a short-pulse laser irradiation. Chai [19] utilized a finite-volume method to calculate transient radiative transfer in a one-dimensional slab. Kim and Guo [20] applied the discrete ordinates method to the transient radiation problems in cylindrical geometries which appear in laser tissue welding and soldering. Boulanger and Charette [21] presented a numerical tool to study short-pulse laser beam interaction with nonhomogeneous matter. Trivedi et al. [22] performed an experimental study for the measurement of temporal reflected and transmitted signals from tissue phantoms with or without inhomogeneities due to short-pulse laser irradiation. They also validated the experimental measurements by solving the two-dimensional transient radiative transfer equation using the discrete ordinates method. Lu and Hsu [23] studied the transient radiation transport in multi-layer media by the reverse Monte Carlo simulation technique. Recently, Mishra et al. [24] presented a general formulation of the transient radiative transfer equation applicable to a 3D Cartesian enclosure, and compared the three commonly used methods, namely, the discrete transfer method, the discrete ordinates method and the finite volume method.

In their semi-analytical numerical study, de Oliveira et al. [25] considered an isotropically scattering medium of slab geometry and expanded the transient intensity of radiation by a truncated series of Laguerre polynomials in the time variable. They obtained a set of time-independent discrete ordinates problems which were then solved by a hybrid method combining the spectral and the Laplace transform methods. Another Laguerre expansion, in time variable, of the radiation intensity is introduced by Hassan et al. [26] for investigating radiative transfer through a semiconductor or a dielectric film using the single relaxation time approximation to the Boltzmann equation. They applied the Galerkin method to solve the resulting time-independent equation.

Okutucu et al. [27] extended the Galerkin technique for the solution of transient radiative transfer in a one-dimensional absorbing and isotropically scattering plane-parallel gray medium irradiated with a short-pulse laser of rectangular profile on one of its boundaries. The authors further applied the method to the transient case of short-pulse Gaussian irradiation [28]. In both cases, the Galerkin method was extended to study the ensuing transient radiative transfer in the medium. In these two applications, the integral form of the radiative transfer equation for the time-dependent source function was transformed into a set of ordinary differential equations for the time-dependent expansion coefficients of the power series expansion of the source function. Although the method proved to be relatively simple to implement and the results agreed well with those available in the literature, it was not found to be efficient in that, at the end, a set of ordinary differential equations had to be solved numerically over the time spectrum. It was, therefore, concluded that the method would not be an effective tool if it were to be extended to study interaction problems.

In the present study, in an effort to improve the effectiveness of the Galerkin method, a new solution technique is developed for a one-dimensional absorbing and isotropically scattering plane-parallel gray medium with short-pulse irradiation of rectangular profile. Following the work of Hassan et al. [26], first, the time-dependent radiative intensity is expanded in a series of Laguerre polynomials with time as the argument. Next, moments of the radiative transfer equation, as well as of the boundary conditions, are taken in accordance with the orthogonality property of the Laguerre polynomials to obtain a set of coupled time-independent radiative transfer problems. This set, in turn, is reduced to a set of algebraic equations by the application of the Galerkin method. The transient transmittance and reflectance of the medium are evaluated and compared with previously obtained results. Recently, the authors have also discussed an approximate version of the method developed here to study transient radiative transfer with short-pulse irradiation of Gaussian profile [29].

It is demonstrated that the Laguerre–Galerkin technique is not only easier to implement and more efficient but also yields more accurate results compared to the direct application of the Galerkin method. The results are in very good agreement with those available in the literature.

2. Formulation

Consider the radiative transfer problem depicted in Fig. 1. The medium is non-emitting, absorbing, isotropically scattering and plane-parallel. It is also considered to have azimuthal symmetry with constant absorption coefficient κ and scattering coefficient σ. The two boundaries at $x = 0$ and $x = L$ are non-reflecting and non-refracting (with refractive index $n = 1$). The boundary at $x = 0$ is irradiated by a collimated rectangular pulse from a direction $\widehat{\mathbf{\Omega}}_0$ (of polar angle θ_0 and azimuthal angle ϕ_0). It can be shown that the intensity $I_d(\tau, \mu, t)$ of the transient diffuse radiation field, which results from the isotropically out-scattered radiation from the collimated component $I_c(\tau, \mu, t)$ travelling through the medium, satisfies the following form of the transient radiative transfer equation [27,28,30]:

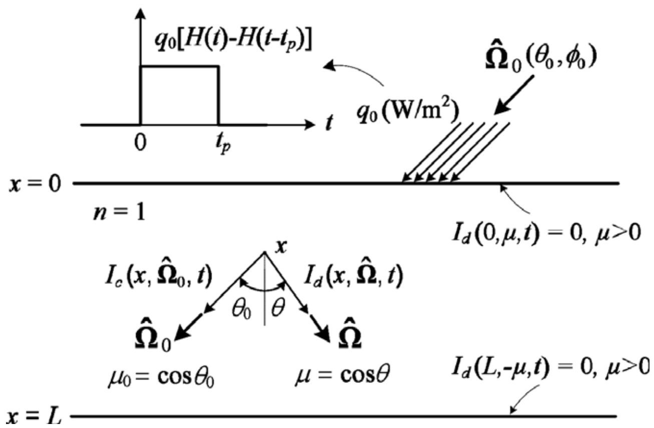

Fig. 1. Schematic representation of boundary conditions.

$$\frac{\partial\psi}{\partial t^*}+2\mu\frac{\partial\psi}{\partial\tau}+\psi(\tau,\mu,t^*)=\frac{\omega}{2}\int_{-1}^{1}\psi(\tau,\mu',t^*)\,\mathrm{d}\mu'+\frac{\omega}{4}\mathrm{e}^{-\tau/2\mu_0}F(\tau,t^*),$$
$$-1\leqslant\mu\leqslant 1,\ 0<\tau<\tau_0 \text{ and } t^*\geqslant\tau \tag{1}$$

with the initial condition $\psi(\tau,\mu,0)=0$ and the two boundary conditions

$$\psi(0,\mu,t^*)=0,\quad \mu>0 \tag{2a}$$
$$\psi(\tau_0,-\mu,t^*)=0,\quad \mu>0 \tag{2b}$$

where $\psi(\tau,\mu,t^*)$ is the dimensionless diffuse intensity defined as

$$\psi(\tau,\mu,t^*)=\frac{I_\mathrm{d}(\tau,\mu,t^*)}{q_0/\pi} \tag{3}$$

In these relations, $t^*=(\kappa+\sigma)ct$ represents the dimensionless time where c is the speed of light in the medium, $\omega=\sigma/(\kappa+\sigma)$ denotes the scattering albedo, $\tau=2(\kappa+\sigma)x$ is the optical variable and $\tau_0=2(\kappa+\sigma)L$ is the optical thickness of the medium. Moreover, q_0 is the total radiative flux of the collimated irradiation through a surface normal to $\widehat{\boldsymbol{\Omega}}_0$.

In Eq. (1), the second term on the right-hand side represents the source for the dimensionless scattered radiation field intensity $\psi(\tau,\mu,t^*)$ due to the isotropically out-scattered radiation from the collimated component in the medium, and the term $F(\tau,t^*)$ is given by

$$F(\tau,t^*)=\left[H\left(t^*-\frac{\tau}{2\mu_0}\right)-H\left(t^*-t_p^*-\frac{\tau}{2\mu_0}\right)\right]=\begin{cases}1, & t^*\in\left[\frac{\tau}{2\mu_0},\frac{\tau}{2\mu_0}+t_p^*\right)\\[4pt] 0, & t^*\notin\left[\frac{\tau}{2\mu_0},\frac{\tau}{2\mu_0}+t_p^*\right)\end{cases} \tag{4}$$

where $t_p^*=(\kappa+\sigma)ct_p$ is the dimensionless pulse width, and $H(t^*)$ represents the Heaviside step function.

The dimensionless intensity $\psi(\tau,\mu,t^*)$ is now expanded in a truncated series of Laguerre polynomials $L_k(t^*)$ as follows:

$$\psi(\tau,\mu,t^*)=\sum_{k=0}^{K}\phi^{(k)}(\tau,\mu)L_k(t^*) \tag{5}$$

Next, the expansion (5) is substituted into Eq. (1). The moments of the resultant relation are then taken by first multiplying it by $\mathrm{e}^{t^*}L_k(t^*)$ and integrating over $t^*\in(0,\infty)$ to yield

$$2\mu\frac{\partial\phi^{(l)}}{\partial\tau}+\phi^{(l)}(\tau,\mu)-\sum_{k=m+1}^{K}\phi^{(k)}(\tau,\mu)=\frac{\omega}{2}\int_{-1}^{1}\phi^{(l)}(\tau,\mu')\,\mathrm{d}\mu'+2F^{(l)}(\tau),\quad l=0,1,2,\ldots,K \tag{6}$$

with

$$F^{(l)}(\tau)=\frac{\omega}{8}\mathrm{e}^{-\tau/2\mu_0}\int_0^\infty \mathrm{e}^{-t^*}L_l(t^*)F(\tau,t^*)\,\mathrm{d}t^* \tag{7}$$

In obtaining Eq. (6), use has been made of the orthogonality property of the Laguerre polynomials $L_k(x)$, $k=0,1,2,\ldots$, given by [31]

$$\int_0^\infty \mathrm{e}^{-x}L_k(x)L_l(x)\,\mathrm{d}x=\delta_{kl} \tag{8}$$

where δ_{kl} is the Kronecker delta, and of the relation [32]

$$\frac{\mathrm{d}L_l(t^*)}{\mathrm{d}t^*}=-\sum_{j=0}^{l-1}L_j(t^*) \tag{9}$$

Noting that $L_k(0)=1$, substitution of the expansion (5) into the initial condition $\psi(\tau,\mu,0)=0$, on the other hand, gives

$$\sum_{k=0}^{K}\phi^{(k)}(\tau,\mu)=0 \tag{10}$$

Combining Eqs. (6) and (10), together with the substitution of the expansion (5) into the boundary conditions (2a) and (2b), now yields the following set of coupled steady-state radiative transfer problems

$$\mu\frac{\partial\phi^{(l)}}{\partial\tau}+\phi^{(l)}(\tau,\mu)=S^{(l)}(\tau),\quad l=0,1,2,\ldots,K$$
$$-1\leqslant\mu\leqslant 1 \text{ and } 0<\tau<\tau_0 \tag{11}$$

$$\phi^{(l)}(0,\mu)=0,\quad \mu>0 \tag{12a}$$
$$\phi^{(l)}(\tau_0,-\mu)=0,\quad \mu>0 \tag{12b}$$

In Eq. (11), the terms $S^{(l)}(\tau)$ represent the source functions and are given by

$$S^{(l)}(\tau)=F^{(l)}(\tau)+\frac{\omega}{2}G^{(l)}(\tau)-\frac{1}{2}\sum_{k=0}^{l-1}G^{(k)}(\tau),\quad l=0,1,2,\ldots,K \tag{13}$$

where the terms $G^{(l)}(\tau)$ are defined as

$$G^{(l)}(\tau) = \frac{1}{2}\int_{-1}^{1} \phi^{(l)}(\tau,\mu')\,\mathrm{d}\mu', \quad l = 0,1,2,\ldots,K \tag{14}$$

which represent the angular averages of the steady-state intensities of the coupled radiation problems defined by Eqs. (11), (12a) and (12b).

In defining the source functions $S^{(l)}(\tau)$ in Eq. (13), the following approximation was introduced:

$$\sum_{k=0}^{l-1} \phi^{(k)}(\tau,\mu) \cong \sum_{k=0}^{l-1} G^{(k)}(\tau) \tag{15}$$

which stipulates that the effects of $\phi^{(k)}(\tau,\mu)$ when $k = 0, 1, 2, \ldots, l-1$ on $\phi^{(l)}(\tau,\mu)$ is the same as the effects of $G^{(k)}(\tau)$, $k = 0,1,2,\ldots,l-1$.

2.1. Formal solutions for $\phi^{(l)}(\tau,\mu)$

Integrating Eq. (11) over the optical variable τ from $\tau = 0$ to any τ for $\mu > 0$, together with the use of the condition (12a), gives

$$\phi^{(l)}(\tau,\mu) = \frac{1}{\mu}\int_0^{\tau} \mathrm{e}^{-(\tau-\tau')/\mu} S^{(l)}(\tau')\,\mathrm{d}\tau', \quad \mu > 0 \tag{16a}$$

Similarly, integrating Eq. (11) from any τ to $\tau = \tau_0$ for $\mu < 0$, together with the use of the condition (12b), yields

$$\phi^{(l)}(\tau,-\mu) = \frac{1}{\mu}\int_{\tau}^{\tau_0} \mathrm{e}^{-(\tau'-\tau)/\mu} S^{(l)}(\tau')\,\mathrm{d}\tau', \quad \mu > 0 \tag{16b}$$

Eqs. (16a) and (16b) are formal solutions for $\phi^{(l)}(\tau,\mu)$. They do not represent a complete solution since both relations are expressed in terms of $S^{(l)}(\tau)$, which in turn depends on $\phi^{(l)}(\tau,\mu)$ as indicated by Eqs. (13) and (14).

2.2. Integral equation for $S^{(l)}(\tau)$

Substitution of the formal solutions (16a) and (16b) into Eq. (14) yields

$$G^{(l)}(\tau) = \frac{1}{2}\int_0^{\tau_0} S^{(l)}(\tau')E_1(|\tau-\tau'|)\,\mathrm{d}\tau', \quad l = 0,1,2,\ldots,K \tag{17}$$

which, when substituted into Eq. (13), gives

$$S^{(l)}(\tau) = F^{(l)}(\tau) - \frac{1}{2}\sum_{k=0}^{l-1} G^{(k)}(\tau) + \frac{\omega}{4}\int_0^{\tau_0} S^{(l)}(\tau')E_1(|\tau-\tau'|)\,\mathrm{d}\tau', \quad l = 0,1,2,\ldots,K \tag{18}$$

where $E_n(x)$ is the exponential integral function.

Eq. (18) represents a set of coupled singular integral equations for the source functions $S^{(l)}(\tau)$. In the following section, following the procedure introduced by Özışık and Yener [33], a Galerkin method of solution is described to obtain $S^{(l)}(\tau)$ from Eq. (18).

3. Solution to $S^{(l)}(\tau)$ by Galerkin method

Following the method introduced in [33], the source functions $S^{(l)}(\tau)$ are first expanded in a power series as

$$S^{(l)}(\tau) = \sum_{n=0}^{N} C_n^{(l)}\tau^n, \quad l = 0,1,2,\ldots,K \tag{19}$$

where $C_n^{(l)}$ are the unknown expansion coefficients to be determined by the application of the Galerkin method. Next, the expansion (19) is substituted into the integral equation (18), and then the resultant equations are integrated over τ from $\tau = 0$ to $\tau = \tau_0$ after they have been multiplied by τ^m. This operation yields, for each $l = 0,1,2,\ldots,K$, the following set of $N+1$ algebraic equations for the determination of the unknown expansion coefficients $C_n^{(l)}$:

$$\sum_{n=0}^{N} B_{mn}C_n^{(l)} = A_m^{(l)}, \quad m = 0,1,2,\ldots,N \tag{20}$$

In Eq. (20), the constants $A_m^{(l)}$ are given by

$$A_m^{(l)} = F_m^{(l)} - \frac{1}{2}\sum_{k=0}^{l-1}\sum_{n=0}^{N} C_n^{(l)}D_{mn}, \quad l = 0,1,2,\ldots,K \tag{21}$$

where

$$F_m^{(l)} = \frac{\omega}{8}2^{m+1}l!\sum_{j=0}^{l}\frac{(-1)^j}{(j!)^2(l-j)!}(I_{mj} - J_{mj}) \tag{22}$$

In Eq. (22), I_{mj} and J_{mj} are defined, respectively, as

$$\begin{aligned} I_{mj} = {} & \mu_0^{m+1}m!\left\{j! - \Gamma\left(1+j,\frac{\tau_0}{2\mu_0}\right) - \sum_{k=0}^{m}\frac{2^{-j-m+k-1}}{(l-k)!}\right. \\ & \left.\times\left[(j+m-k)! - \Gamma\left(j+m-k+1,\frac{\tau_0}{\mu_0}\right)\right]\right\} \\ & + £_m\left(\frac{\tau_0}{2}\right)\Gamma\left(1+j,\frac{\tau_0}{2\mu_0}\right) \end{aligned} \tag{23}$$

and

$$\begin{aligned} J_{mj} = {} & \mu_0^{m+1}m!\left\{\Gamma(1+j,t_p^*) - \Gamma\left(1+j,t_p^*+\frac{\tau_0}{2\mu_0}\right)\right. \\ & - \mathrm{e}^{t_p^*}\sum_{k=0}^{m}\sum_{r=0}^{m-k}\frac{(-1)^r(t_p^*)^r 2^{k+r-m-j-1}}{r!(m-k-r)!}\left[\Gamma(m-k-r+j+1,2t_p^*)\right. \\ & \left.\left. - \Gamma\left(m-k-r+j+1,2t_p^*+\frac{\tau_0}{\mu_0}\right)\right]\right\} \\ & + £_m\left(\frac{\tau_0}{2}\right)\Gamma\left(1+j,t_p^*+\frac{\tau_0}{2\mu_0}\right) \end{aligned} \tag{24}$$

where $£_m(\xi)$ is defined as and given by

$$\begin{aligned} £_m(\xi) &= \int_0^{\xi} \mathrm{e}^{-u/\mu_0}u^m\,\mathrm{d}u \\ &= \mu_0^{m+1}m! - \mathrm{e}^{-\xi/\mu_0}\sum_{k=0}^{m}\frac{m!}{(m-k)!}\xi^{m-k}\mu_0^{k+1} \end{aligned} \tag{25}$$

and $\Gamma(n,x)$ represents *the upper incomplete Gamma function*. Furthermore, in Eq. (20), the constants B_{mn} are given by

$$B_{mn} = \frac{\tau_0^{m+n+1}}{m+n+1} - \frac{\omega}{2} D_{mn} \tag{26}$$

where

$$D_{mn} = \frac{1}{2} m!n! \left\{ \frac{(-1)^{n+1}}{m+n+2} + (-1)^n \sum_{i=0}^{m} \frac{\tau_0^{m-i}}{(m-i)!} E_{n+i+3}(\tau_0) + (-1)^m \sum_{i=0}^{n} \frac{\tau_0^{n-i}}{(n-i)!} E_{m+i+3}(\tau_0) + \frac{1}{m!} \sum_{i=0}^{n} \frac{1}{(n-i)!} \frac{1+(-1)^i}{i+1} \frac{\tau_0^{m+n-i+1}}{m+n-i+1} - \sum_{i=0}^{n} \sum_{j=0}^{m} \frac{1}{(n-i)!(m-j)!} \frac{(-1)^j}{i+j+2} \tau_0^{m+n-i-j} \right\} \tag{27}$$

4. Local radiative flux

The local radiative flux is given in dimensionless form by [30]

$$Q(\tau, t^*) = \frac{q(\tau, t^*)}{q_0} = Q^+(\tau, t^*) - Q^-(\tau, t^*) \tag{28}$$

where $q(\tau, t^*)$ is the local radiative flux in the medium, and the dimensionless forward and backward radiative fluxes $Q^{\pm}(\tau, t^*)$ are given by

$$Q^+(\tau, t^*) = \mu_0 \mathrm{e}^{-\tau/(2\mu_0)} F(\tau, t^*) + 2 \int_0^1 \psi(\tau, \mu, t^*) \mu \, \mathrm{d}\mu, \quad t^* \geqslant \tau/2 \tag{29a}$$

$$Q^-(\tau, t^*) = 2 \int_0^1 \psi(\tau, -\mu, t^*) \mu \, \mathrm{d}\mu, \quad t^* \geqslant \tau/2 \tag{29b}$$

In Eq. (29a), the first term on the right-hand side represents the contribution to the forward flux at location τ and time t^* of the attenuating collimated component, whereas the second term is the contribution of the scattered radiation field. For the backward flux, the only contribution comes from the scattered radiation field as indicated in Eq. (29b).

5. Results and discussion

The boundary at $x = 0$ is considered to be irradiated perpendicularly (i.e., $\mu_0 = 1$) with a short-pulse collimated irradiation of rectangular profile. The transient transmittance and reflectance of the medium are obtained for different values of the optical thickness τ_0, scattering albedo ω and the pulse width t_p^*. The transient transmittance is defined as the net dimensionless radiative flux emerging out of the medium, and in the problem under consideration it is the net forward dimensionless radiative flux at $\tau = \tau_0$; that is

$$Q^+(\tau_0, t^*) = \mu_0 \mathrm{e}^{-\tau_0/(2\mu_0)} F(\tau_0, t^*) + 2 \sum_{k=0}^{K} L_k(t^*) \sum_{n=0}^{N} C_n^{(k)} n! \times \left\{ (-1)^{n+1} E_{n+3}(\tau_0) + \sum_{\alpha=0}^{n} \frac{(-1)^\alpha}{(n-\alpha)!(\alpha+2)} \tau_0^{n-\alpha} \right\}, \quad t^* \geqslant \tau_0/2 \tag{30}$$

Similarly, the reflectance, which is the net radiative flux from the boundary at $\tau = 0$, is

$$Q^-(0, t^*) = 2 \sum_{k=0}^{K} L_k(t^*) \sum_{n=0}^{N} C_n^{(k)} n! \left\{ \frac{1}{n+2} - \sum_{\alpha=0}^{n} \frac{\tau_0^{n-\alpha}}{(n-\alpha)!} E_{\alpha+3}(\tau_0) \right\}, \quad t^* \geqslant 0 \tag{31}$$

It should be noted that the coefficients $C_n^{(k)}$ of the power series expansion (19) are constants, hence, can easily be evaluated from the solution of the set of $N+1$ algebraic equations (20) for each $k = 0, 1, 2, \ldots, K$. A Gauss elimination algorithm with pivoting is utilized to obtain the coefficients. Once they are calculated, the transmittance, $Q^+(\tau_0, t^*)$, and the reflectance, $Q^-(0, t^*)$, of the medium are readily obtained from Eqs. (30) and (31).

Calculations were performed in the C programming environment to evaluate the transient behavior of both transmittance and reflectance for various values of the optical thickness, pulse duration and scattering albedo. The dimensionless time step was taken as $\Delta t^* = 0.001$, which was found to be the optimum value below which no significant improvement in the results was observed.

For small to moderate optical thicknesses, convergence was reached with as few as five terms in the truncated power series expansion in optical thickness, and 60 terms in the truncated Laguerre expansion in time. It was decided that convergence was achieved when the addition of more terms did not change the value of the dimensionless transmittance or reflectance by more than 10^{-5}.

Fig. 2 illustrates the effect of the optical thickness on transmittance for $t_p^* = 1$ and $\omega = 0.998$. The time at which

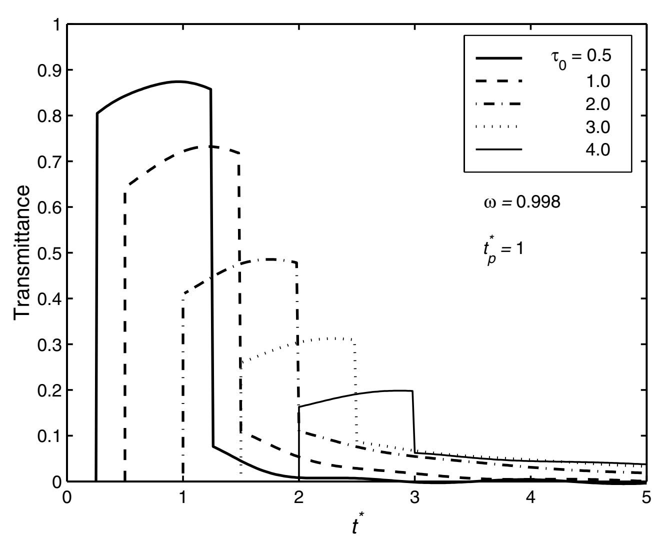

Fig. 2. Effect of optical thickness on transmittance for $N = 4$, $K = 59$.

the first transmitted radiation is observed at τ_0 depends on the optical thickness of the medium. It takes $t^* = \tau_0/2$ for the collimated component to traverse the medium. Once the collimated signal exits the medium at $t^* = t_p^* + \tau_0/2$, a sudden decrease in the transmitted flux is observed. As expected, the peak value of the transmittance is higher for optically thinner media due to stronger collimated component that exits the medium. The decay rate beyond $t^* = t_p^* + \tau_0/2$ decreases with increasing optical thickness since it takes longer time for the scattered signals to exit the medium in the optically thicker cases.

The corresponding reflectance behavior is presented in Fig. 3 again for $t_p^* = 1$ and $\omega = 0.998$. The reflected radiation is observed as soon as the irradiation hits the boundary at $\tau = 0$. The dependence of the peak value on the optical thickness is not as strong as that for transmittance. On the other hand, as the optical thickness increases, the backscattered signal is observed for a longer period of time. It is also noted from Fig. 3 that the reflectance curves get closer to each other as the optical thickness increases. It is also observed that the stability of the results are better for $\tau_0 \leqslant 3$.

The effect of the pulse duration on the transient behavior of transmittance for $t_p^* = 0.1, 1$ and 2 is shown for $\omega = 0.998$ in Fig. 4. For the case of $t_p^* = 0.1$, a smaller dimensionless time step of 0.0001 was used for better convergence. As the pulse duration becomes comparable to or larger than the optical thickness, a smoother transmittance curve is obtained. The strength and duration of the transmitted signal increases with increasing pulse length, as expected. The dependence of reflectance on the pulse duration is given in Fig. 5.

In Fig. 6, the effect of the scattering albedo on transmittance is illustrated for a medium with an optical thickness of $\tau_0 = 2$ and for $t_p^* = 1$. It is shown that the transmitted signal is available for longer periods of time, and its peak value is higher for increased scattering albedo values. As

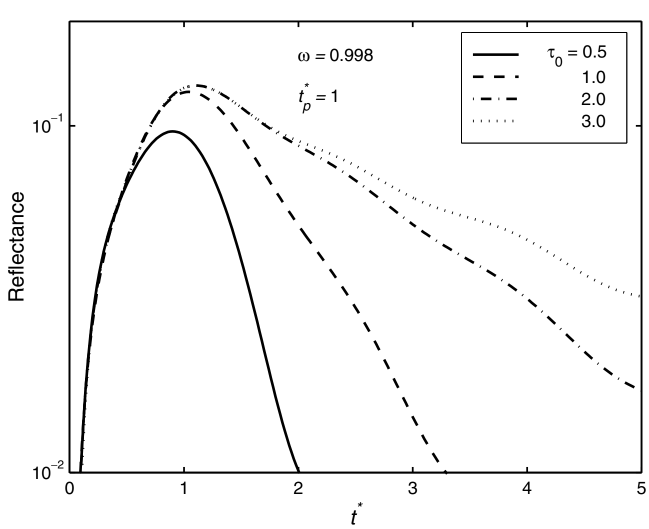

Fig. 3. Effect of optical thickness on reflectance for $N = 4$, $K = 59$.

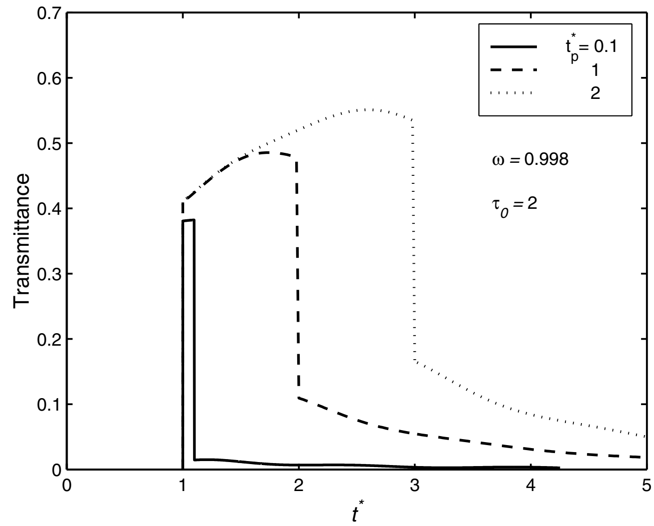

Fig. 4. Effect of pulse duration on transmittance for $N = 4$, $K = 59$.

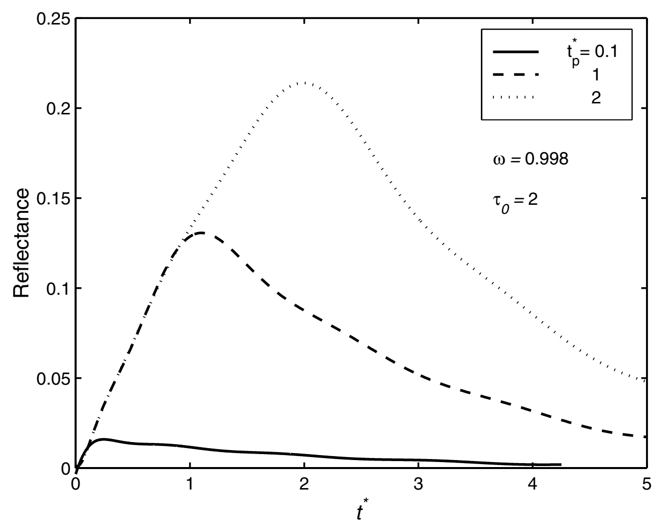

Fig. 5. Effect of pulse duration on reflectance for $N = 4$, $K = 59$.

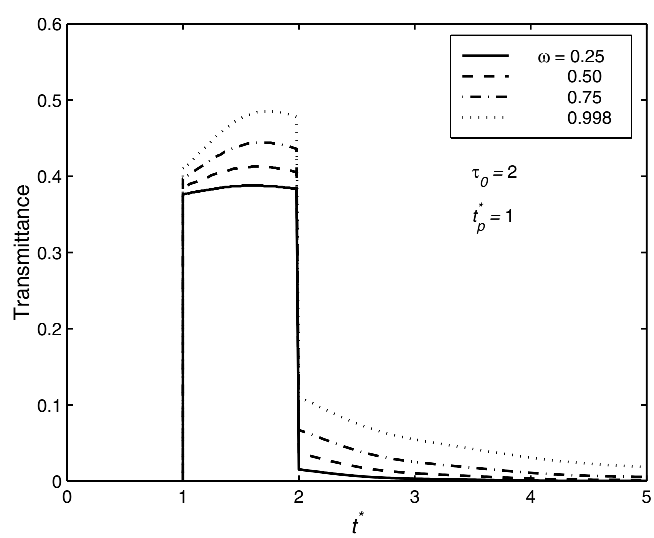

Fig. 6. Effect of scattering albedo on transmittance for $N = 4$, $K = 59$.

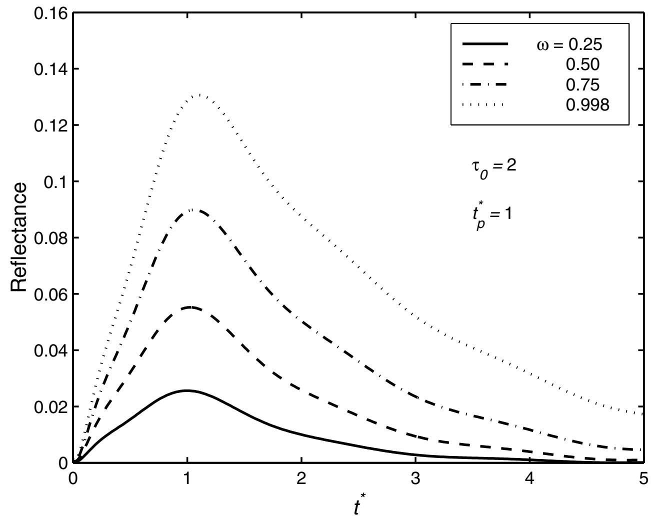

Fig. 7. Effect of scattering albedo on reflectance for $N = 4$, $K = 59$.

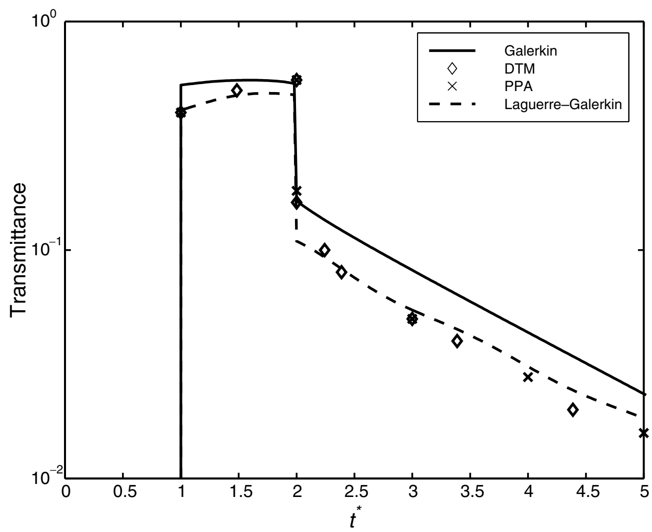

Fig. 8. Comparison of transient transmittance results obtained by Laguerre–Galerkin, Galerkin, discrete transfer (DTM) and piecewise parabolic advection (PPA) scheme along with discrete ordinates method for $\tau_0 = 2$, $t_p^* = 1$ and $\omega = 0.998$.

indicated in Fig. 7, the reflectance of the medium shows a similar behavior; that is, a higher peak value and a longer duration of the signal at a higher value of the scattering albedo.

The results are also compared with those obtained by the discrete transfer and the piecewise parabolic advection methods [18] and an excellent agreement is reached. Figs. 8 and 9 compare the transmittance and reflectance results, respectively, for $\tau_0 = 2$, $t_p^* = 1$ and $\omega = 0.998$.

6. Conclusions

An improved solution technique has been developed for the transient radiative transfer problem in a participating

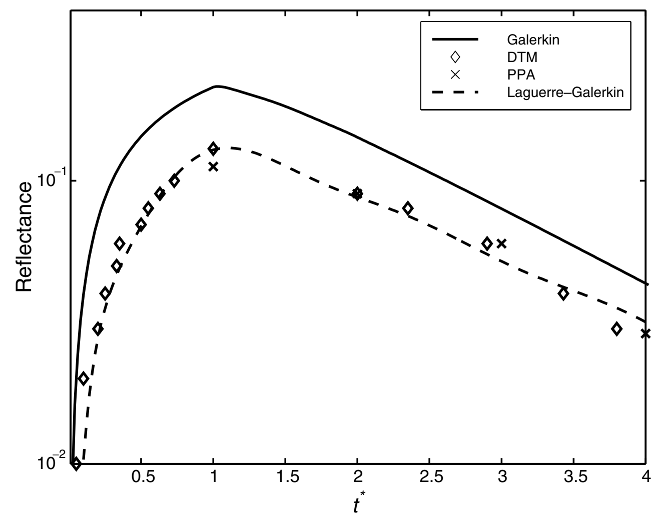

Fig. 9. Comparison of transient reflectance results obtained by Laguerre–Galerkin, Galerkin, discrete transfer (DTM) and piecewise parabolic advection (PPA) scheme along with discrete ordinates method for $\tau_0 = 2$, $t_p^* = 1$ and $\omega = 0.998$.

medium one boundary of which is exposed to short-pulse irradiation of rectangular profile. In this technique, the time-dependent dimensionless radiative intensity within the one-dimensional, absorbing, non-emitting and isotropically scattering medium is expanded in a series of Laguerre polynomials with time as the argument. The moments of the radiative transfer equation, as well as of the boundary conditions, are then taken in accordance with the orthogonality property of the Laguerre polynomials, yielding a set of coupled time-independent radiative transfer problems. This set, in turn, is reduced to a set of algebraic equations utilizing the Galerkin method. The transient transmittance and reflectance are re-evaluated and compared with previously obtained ones. It has been demonstrated that this approach is not only more efficient but also yields results that agree very well with those available in the literature. More accurate results have been obtained compared to the direct application of the Galerkin method. It should also be noted that the method works better for small to moderate optical thicknesses where the hyperbolic approximations reported in the literature seem to fail.

References

[1] S. Kumar, K. Mitra, Transient radiative transfer, in: P.M. Mengüç (Ed.), Radiative Transfer-I, Proceedings of the First International Symposium on Radiation Transfer, ICHMT, Begell House, 1996, pp. 488–504.
[2] S. Kumar, K. Mitra, Y. Yamada, Hyperbolic damped-wave models for transient light-pulse propagation in scattering media, Appl. Opt. 35 (1996) 3372–3378.
[3] K. Mitra, M.-S. Lai, S. Kumar, Transient radiation transport in participating media within a rectangular enclosure, J. Thermophys. Heat Transfer 11 (3) (1997) 409–414.
[4] S. Kumar, K. Mitra, Microscale aspects of thermal radiation transport and laser applications, Adv. Heat Transfer 33 (1999) 187–294.

[5] K. Mitra, S. Kumar, Development and comparison of models for light-pulse transport through scattering–absorbing media, Appl. Opt. 38 (1) (1999) 188–196.

[6] M. Sakami, K. Mitra, P.-F. Hsu, Analysis of light pulse transport through two-dimensional scattering and absorbing media, in: P.M. Mengüç, N. Selçuk (Eds.), Radiative Transfer-III, Proceedings of the Third International Symposium on Radiation Transfer, ICHMT, Begell House, 2001, pp. 44–52.

[7] G.C. Pomraning, The Equations of Radiation Hydrodynamics, Pergamon Press, Oxford, 1973.

[8] C.-Y. Wu, S.-H. Wu, Integral equation formulation for transient radiative transfer in an anisotropically scattering medium, Int. J. Heat Mass Transfer 43 (2000) 2009–2020.

[9] S.-H. Wu, C.-Y. Wu, Integral equation solutions for transient radiative transfer in nonhomogeneous anisotropically scattering media, J. Heat Transfer 122 (2000) 818–822.

[10] C.-Y. Wu, Propagation of scattered radiation in a participating planar medium with pulse irradiation, J. Quant. Spectrosc. Radiat. Transfer 64 (2000) 537–548.

[11] Z.-M. Tan, P.-F. Hsu, An integral formulation of transient radiative transfer, J. Heat Transfer 123 (2001) 466–475.

[12] Z.-M. Tan, P.-F. Hsu, Transient radiative transfer in three-dimensional homogeneous and nonhomogeneous participating media, in: P. Mengüç, N. Selçuk (Eds.), Radiative Transfer-III, Proceedings of the Third International Symposium on Radiation Transfer, ICHMT, Begell House, 2001, pp. 54–66.

[13] Z. Guo, S. Kumar, Radiation element method for transient hyperbolic radiative transfer in plane-parallel inhomogeneous media, Numer. Heat Transfer B 39 (2001) 371–387.

[14] S. Chandrasekhar, Radiative Transfer, Oxford University Press, Oxford, 1950.

[15] C.-Y. Wu, N.-R. Ou, Differential approximations for transient radiative transfer through a participating medium exposed to collimated irradiation, J. Quant. Spectrosc. Radiat. Transfer 73 (2002) 111–120.

[16] Z. Guo, J. Aber, B.A. Garetz, S. Kumar, Pulse laser radiation transfer: Monte Carlo simulation and comparison with experiment, in: P.M. Mengüç, N. Selçuk (Eds.), Radiative Transfer-III, Proceedings of the Third International Symposium on Radiation Transfer, ICHMT, Begell House, 2001, pp. 34–43.

[17] Z. Guo, J. Aber, B.A. Garetz, S. Kumar, Monte Carlo simulation and experiments of pulsed radiative transfer, J. Quant. Spectrosc. Radiat. Transfer 73 (2002) 159–168.

[18] P. Rath, S.C. Mishra, P. Mahanta, U.K. Saha, K. Mitra, Discrete transfer method applied to transient radiative transfer problems in participating medium, Numer. Heat Transfer A 44 (2003) 183–197.

[19] J. Chai, One-dimensional transient radiation heat transfer modeling using a finite-volume method, Numer. Heat Transfer B 44 (2003) 187–208.

[20] K. Kim, Z. Guo, Ultrafast radiation heat transfer in laser tissue welding and soldering, Numer. Heat Transfer A 46 (2004) 23–40.

[21] J. Boulanger, A. Charette, Reconstruction optical spectroscopy using transient radiative transfer equation and pulsed laser: a numerical study, J. Quant. Spectrosc. Radiat. Transfer 93 (2005) 325–336.

[22] A. Trivedi, S. Basu, K. Mitra, Temporal analysis of reflected optical signals for short pulse laser interaction with nonhomogeneous tissue phantoms, J. Quant. Spectrosc. Radiat. Transfer 93 (2005) 337–348.

[23] X. Lu, P.-F. Hsu, Reverse Monte Carlo simulations of light pulse propagation in nonhomogeneous media, J. Quant. Spectrosc. Radiat. Transfer 93 (2005) 349–367.

[24] S.C. Mishra, P. Chugh, P. Kumar, K. Mitra, Development and comparison of the DTM, the DOM and the FVM formulations for the short-pulse laser transport through a participating medium, Int. J. Heat Mass Transfer 49 (2006) 1820–1932.

[25] J.V.P. de Oliveira, A.V. Cardona, M.T. Vilhena, R.C. Barros, A semi-analytical numerical method for time-dependent radiative transfer problems in slab geometry with coherent isotropic scattering, J. Quant. Spectrosc. Radiat. Transfer 73 (2002) 55–62.

[26] T. Hassan, K.R. Naqvi, S.A. El-Wakil, E.M. Abulwafa, Transient radiative heat transfer through thin films using Laguerre–Galerkin method, J. Phys. D 36 (2003) 3014–3026.

[27] T. Okutucu, Y. Yener, A.A. Busnaina, Transient radiative transfer in participating media with pulse-laser irradiation – an approximate Galerkin solution, J. Quant. Spectrosc. Radiat. Transfer 103 (2007) 118–130.

[28] T. Okutucu, Y. Yener, Radiative transfer in participating media with collimated short-pulse Gaussian irradiation, J. Phys. D 39 (2006) 1976–1983.

[29] T. Okutucu, Y. Yener, Transient radiative transfer in participating media with short-pulse irradiation – an approximate Laguerre–Galerkin solution, in: Proceedings of the 13th International Heat Transfer Conference, Sydney, Australia, August 2006.

[30] T. Okutucu, Analytical and experimental study of transient radiative transfer in participating media with short-pulse laser irradiation, Ph.D. Thesis, Northeastern University, Boston, Massachusetts, USA, August 2005.

[31] M. Abramowitz, I.A. Stegun, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, Ninth Dover Printing, 10th GPO Printing Edition, Dover, New York, 1964.

[32] E.W. Weisstein, Laguerre Polynomial, From Mathworld – A Wolfram Web Resource. http://mathworld.wolfram.com/LaguerrePolynomial.html, 2004.

[33] M.N. Özışık, Y. Yener, The Galerkin method for solving radiation transfer in plane-parallel participating media, J. Heat Transfer 104 (1982) 351–354.